Name solutions

March 6, 2013

ECE 311 Exam 2 Spring 2013

Closed Text and Notes

1) Be sure you have 10 pages plus the two-sided equation sheet.

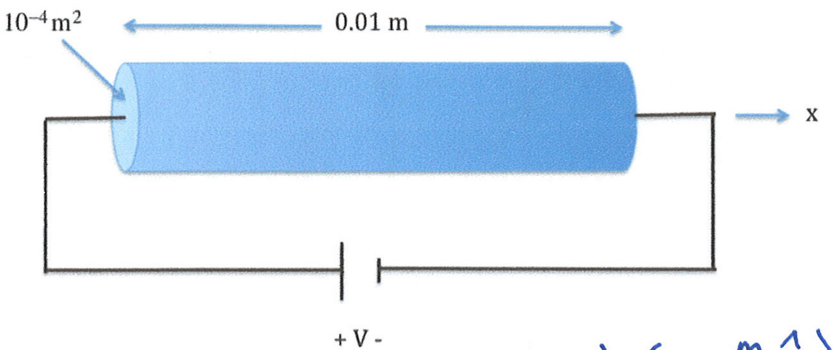
2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(5 pts) 1. In semiconductors you can have current flow due to positively charged holes. A hole has a charge of $+1\text{x}10^{-19}\,\text{C}$. A silicon rod has length 0.01 m and cross-sectional area $1\text{x}10^{-4}\,\text{m}^2$. The hole density in the silicon is $1\text{x}10^{17}\,\dfrac{\text{holes}}{\text{m}^3}$. A voltage is placed across the length of the silicon rod and the holes move with average velocity $100\dfrac{\text{m}}{\text{s}}\,\hat{\mathbf{a}}_x$. What current is flowing in the silicon rod?



$$\rho = n e$$

$$\rho = \left(10^{17}\frac{\text{holes}}{m^3}\right)\left(1.6\times10^{-19}\frac{C}{\text{hole}}\right)$$

$$\rho = 1.6\times10^{-2}\,\frac{C}{m^3}$$

$$\vec{J} = \rho\vec{u} = \left(1.6\times10^{-2}\frac{C}{m^3}\right)\left(100\frac{m}{s}\hat{a}_x\right) = 1.6\frac{C}{sm^2}\hat{a}_x = 1.6\frac{A}{m^2}\hat{a}_x$$

$$I = JS = \left(1.6\frac{A}{m^2}\right)\left(10^{-4}\,m^2\right)$$

$$I = 1.6\times10^{-4}\,A$$

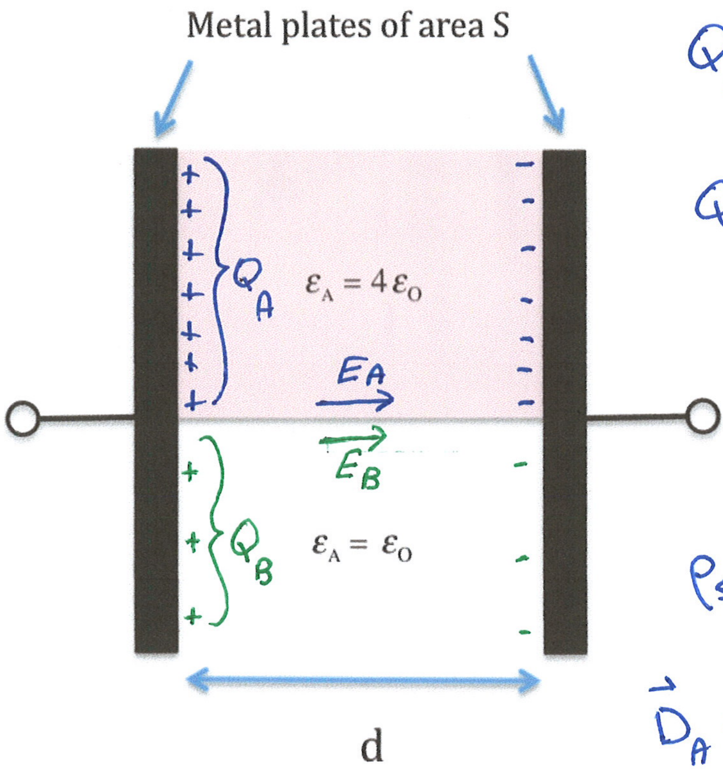
(5 pts) 2. A 1,000 Ω resistor has a length of 0.01 m and a cross-sectional area of $1\text{x}10^{-5}\,\text{m}^2$. What is the conductivity of the material comprising the resistor?

$$R = \rho_c\frac{L}{S} = \frac{1}{\sigma}\frac{L}{S}$$

$$\sigma = \frac{L}{RS} = \frac{0.01\,m}{(10^3\,\Omega)(10^{-5}\,m^2)} = 1\,\frac{1}{\Omega m}$$

$$\sigma = 1\,\frac{S}{m}$$

(15 pts) 3. Find an expression for the capacitance of the parallel plate capacitor shown. The plate separation is d, the area of the plates is S, the top half of the capacitor is filled with a dielectric of permittivity $4\varepsilon_O$ and the bottom half is free space, ε_O. To receive credit you must use a fundamental approach. DO NOT approach the problem as two capacitors in parallel.



Q_A = charge on the positive plate next to dielectric A

Q_B = charge on the positive plate next to the free space region

$$Q = Q_A + Q_B$$

$$\vec{E}_A = \vec{E}_B$$

$$\rho_{sA} = \frac{Q_A}{(S/2)} = \frac{2Q_A}{S} \qquad \rho_{sB} = \frac{Q_B}{(S/2)} = \frac{2Q_B}{S}$$

$$\vec{D}_A = \rho_{sA}\hat{a}_x = \frac{2Q_A}{S}\hat{a}_x \qquad \vec{D}_B = \frac{2Q_B}{S}\hat{a}_x$$

$$\vec{E}_A = \frac{2Q_A}{4\epsilon_o S}\hat{a}_x \qquad \vec{E}_B = \frac{2Q_B}{\epsilon_o S}\hat{a}_x$$

$$\vec{E}_A = \vec{E}_B$$

$$\frac{2Q_A}{4\epsilon_o S} = \frac{2Q_B}{\epsilon_o S}$$

$$Q_A = 4Q_B$$

$$Q = 4Q_B + Q_B$$

$$Q = 5Q_B$$

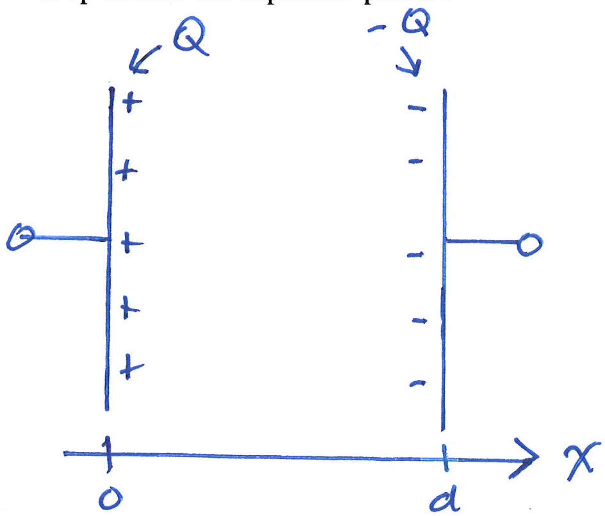
$$V = -\int_d^0 \vec{E}_B \cdot dx\hat{a}_x$$

$$= -\int_d^0 \frac{2Q_B}{\epsilon_o S}dx = \frac{2Q_B d}{\epsilon_o S}$$

$$C = \frac{Q}{V} = \frac{5Q_B}{\left(\frac{2Q_B d}{\epsilon_o S}\right)}$$

$$\boxed{C = \frac{5\epsilon_o S}{2d}}$$

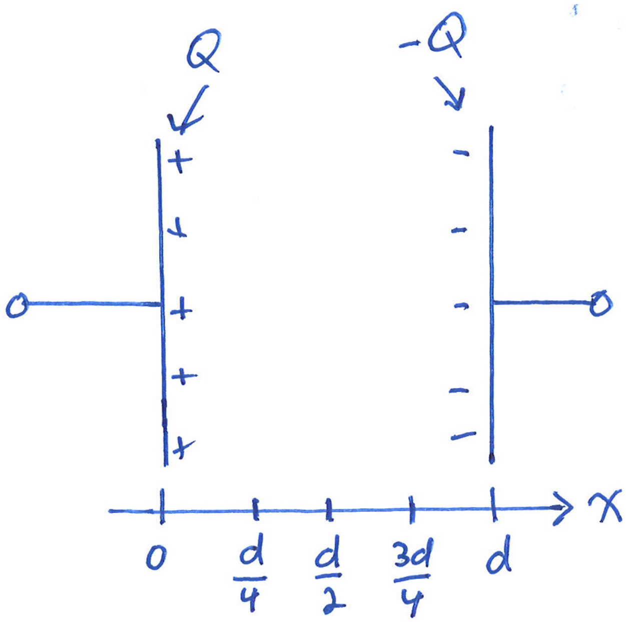
(15 pts) 4. A 60 V battery is connected to a parallel plate capacitor with just air between the plates. The battery is removed without disturbing the charge on the plates. A dielectric with relative dielectric constant of 3 and thickness d/2 is lowered between the plates. What is the potential drop across the capacitor plates?



$$\vec{D} = \frac{Q}{S}\hat{a}_x, \quad 0 < x < d$$

$$\vec{E} = \frac{Q}{\epsilon_0 S}\hat{a}_x, \quad 0 < x < d$$

$$V_{before} = 60V = \frac{Q}{\epsilon_0 S} d$$



$$\vec{D} = \frac{Q}{S}\hat{a}_x, \quad 0 < x < d$$

$$\vec{E} = \frac{Q}{\epsilon_0 S}\hat{a}_x, \quad 0 < x < \frac{d}{4}$$

$$= \frac{Q}{3\epsilon_0 S}\hat{a}_x, \quad \frac{d}{4} < x < \frac{3d}{4}$$

$$= \frac{Q}{\epsilon_0 S}\hat{a}_x, \quad \frac{3d}{4} < x < d$$

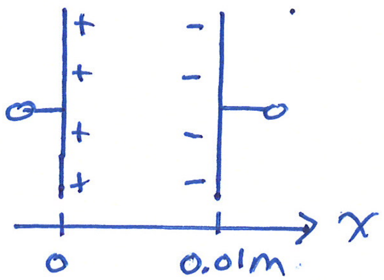
$$V_{after} = -\int_d^0 \vec{E}\cdot d\vec{l} = -\int_d^{3d/4} \frac{Q}{\epsilon_0 S}dx - \int_{3d/4}^{d/4} \frac{Q}{3\epsilon_0 S}dx - \int_{d/4}^{0} \frac{Q}{\epsilon_0 S}dx$$

$$= -\frac{Q}{\epsilon_0 S}x\Big|_d^{3d/4} - \frac{Q}{3\epsilon_0 S}x\Big|_{3d/4}^{d/4} - \frac{Q}{\epsilon_0 S}x\Big|_{d/4}^{0}$$

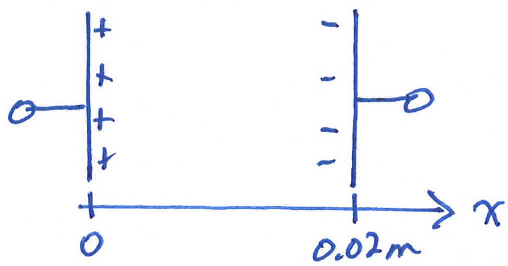
$$= \frac{Qd}{4\epsilon_0 S} + \frac{Qd}{6\epsilon_0 S} + \frac{Qd}{4\epsilon_0 S} = \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{4}\right)\frac{Qd}{\epsilon_0 S} = \frac{2}{3}\frac{Qd}{\epsilon_0 S}$$

$$= \frac{2}{3}V_{before} = \frac{2}{3}(60V) = \boxed{40V = V_{after}}$$

(6 pts). 5. A 90 V battery is connected to a parallel plate capacitor with just air between the plates. The battery is removed without disturbing the charge on the plates. The spacing between the plates is 0.01 m. Without disturbing the charge on the plates, the plates are pulled apart to a separation of 0.02 m. What is the potential across the plates after pulling them apart?



$$V_{before} = -\int_{0.01m}^{0} \vec{E}\cdot d\vec{l} = -Ex\Big|_{0.01m}^{0} = (0.01)E = 90V$$



E does not change because Q does not change and the dielectric (free space) is the same between the plates

$$V_{after} = -\int_{0.02m}^{0} \vec{E}\cdot d\vec{l} = -Ex\Big|_{0.02m}^{0} = (0.02)E$$

so $V_{after} = 2V_{before} = 180V$

(7 pts) 6. Fill in the table with the units, and whether a vector or scalar, for indicated quantities.

	units	Vector or scalar?
Electric Field Intensity	V/m	vector
Electric flux density	C/m^2	vector
Electric flux	C	scalar
Polarization	C/m^2	vector
potential	V = J/C	scalar
Capacitance	F = C/V	scalar
Magnetic Field Intensity	A/m	vector

(6 pts) 7. A linear, isotropic dielectric consists of $10^{22}\,\frac{\text{non-polar atoms}}{\text{m}^3}$. When the electric field $\mathbf{E} = 36\pi \times 10^4\,\frac{\text{V}}{\text{m}}\,\hat{\mathbf{a}}_x$ is applied to this dielectric, all the atoms can be modeled as electric dipoles with electric dipole moment $\mathbf{p} = 10^{-27}\ \text{Cm}\,\hat{\mathbf{a}}_x$. What is the dielectric constant of the dielectric? Note $\varepsilon_o = \frac{10^{-9}}{36\pi}\frac{\text{F}}{\text{m}}$

$$\vec{P} = \frac{\text{dipole moment}}{\text{unit volume}} = n\vec{p} = \left(10^{22}\,\frac{\text{atoms}}{m^3}\right)\left(10^{-27}\,\text{Cm}\,\hat{a}_x\right)$$

$$\vec{P} = 10^{-5}\,\frac{C}{m^2}\,\hat{a}_x$$

$$\vec{P} = \chi_e \epsilon_o \vec{E}$$

$$\chi_e = \frac{P}{\epsilon_o E} = \frac{10^{-5}\,\frac{C}{m^2}}{\left(\frac{10^{-9}}{36\pi}\,\frac{F}{m}\right)\left(36\pi \times 10^4\,\frac{V}{m}\right)} = 1\,\frac{C}{VF} = 1$$

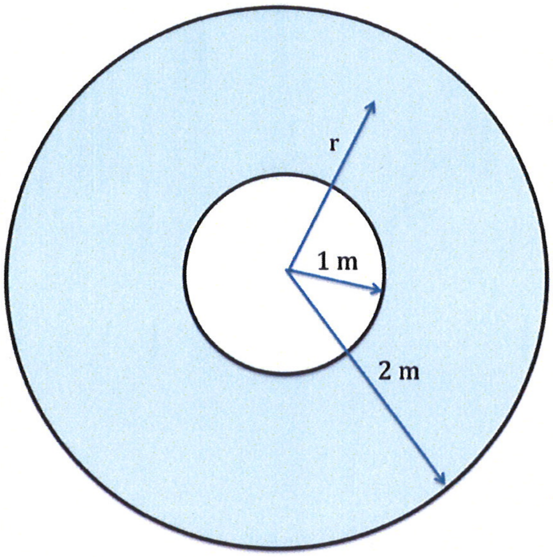
$$\vec{D} = \epsilon_o \vec{E} + \vec{P} = \epsilon_o \vec{E} + \chi_e \epsilon_o \vec{E} = \epsilon_o \underbrace{(1+\chi_e)}_{\epsilon_r} \vec{E}$$

$$= \epsilon_o \epsilon_r \vec{E}$$

$$\epsilon_r = 1 + \chi_e = 1 + 1 = 2$$

$$\boxed{\epsilon_r = 2}$$

(10 pts) 8. Shown are two co-centric conducting spheres. The dielectric material between the spheres has permittivity of $3\varepsilon_0$, A 10 V battery is across the spheres with V(1m) = 10 V and V (2 m) = 0. Using Laplace's equation, find the potential for 1m < r < 2 m.



$\rho_v = 0$ for $1m < r < 2m$

The potential will have spherical symmetry and only depend on r

$V(r)$, so $\frac{\partial V}{\partial \theta} = \frac{\partial V}{\phi} = 0$

so we need to solve Laplace's equation

$\nabla^2 V = 0$ with boundary conditions

$V(1m) = 10V$ and $V(2m) = 0$

In spherical coordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\underbrace{\left(\sin\theta \frac{\partial V}{\partial \theta}\right)}_{=0} + \frac{1}{r^2 \sin^2\theta}\underbrace{\frac{\partial^2 V}{\partial \phi^2}}_{=0} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = A$$

$$\partial V = A r^{-2} dr$$

$$V(r) = -\frac{A}{r} + B$$

apply boundary condition

$$V(2m) = -\frac{A}{2} + B = 0 \Rightarrow B = \frac{A}{2}$$

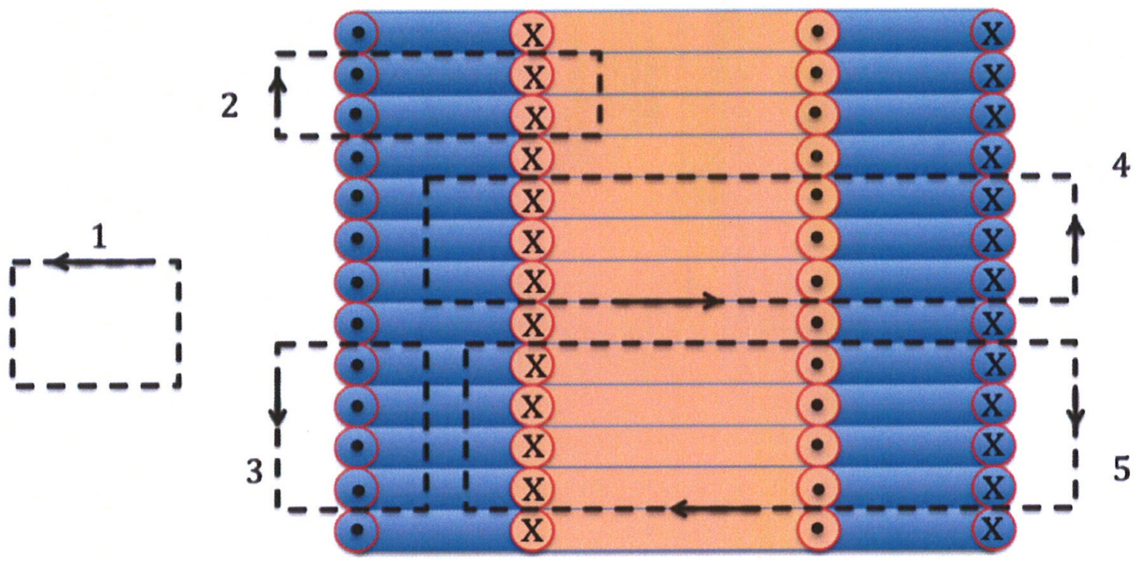
$$V(r) = -\frac{A}{r} + \frac{A}{2}$$

$$V(1m) = -\frac{A}{1} + \frac{A}{2} = 10V \Rightarrow A = -20V$$

$$B = -10V$$

$$\boxed{V(r) = \left(\frac{20}{r} - 10\right) V}$$

(10 pts) 9. Two solenoids have the same number of turns per unit length. They have different diameters and are co-axial. Shown is a cut through the length of the two solenoids indicating the direction of current flow where the same current, 1 A, is flowing in each solenoid. For the paths shown, which are in the same plane as the cut shown through the solenoid, evaluate the following integrals,



$$\oint_1 \mathbf{H} \bullet d\mathbf{l} = 0$$

$$\oint_2 \mathbf{H} \bullet d\mathbf{l} = 0$$

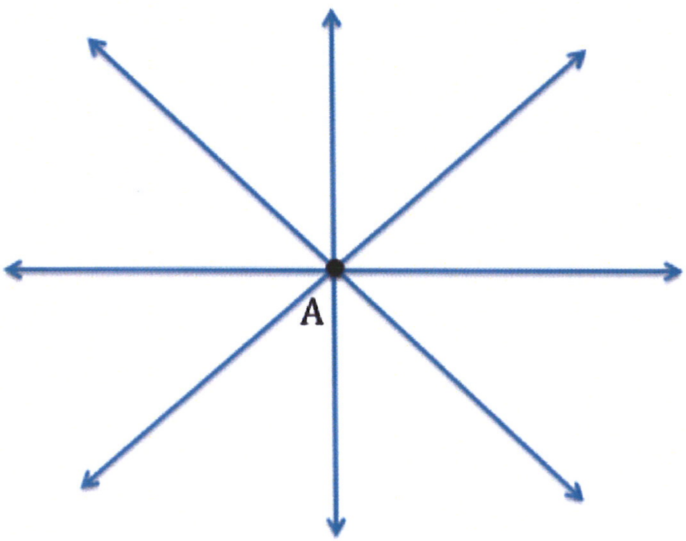
$$\oint_3 \mathbf{H} \bullet d\mathbf{l} = 4A$$

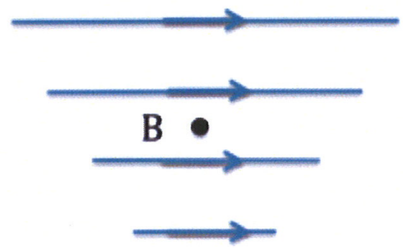
$$\oint_4 \mathbf{H} \bullet d\mathbf{l} = -3A$$

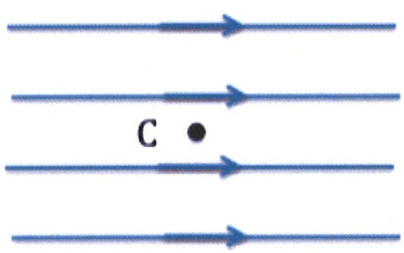
$$\oint_5 \mathbf{H} \bullet d\mathbf{l} = 4A$$

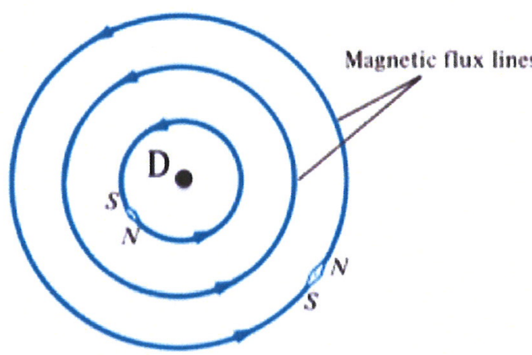
(12 pts) 10. For the vector field shown, indicate whether the curl of the vector field will be positive into-the-page, positive out-of-the-page, or zero at the points indicated.

point	curl
A	0
B	positive into the page
C	0
D	positive out of the page

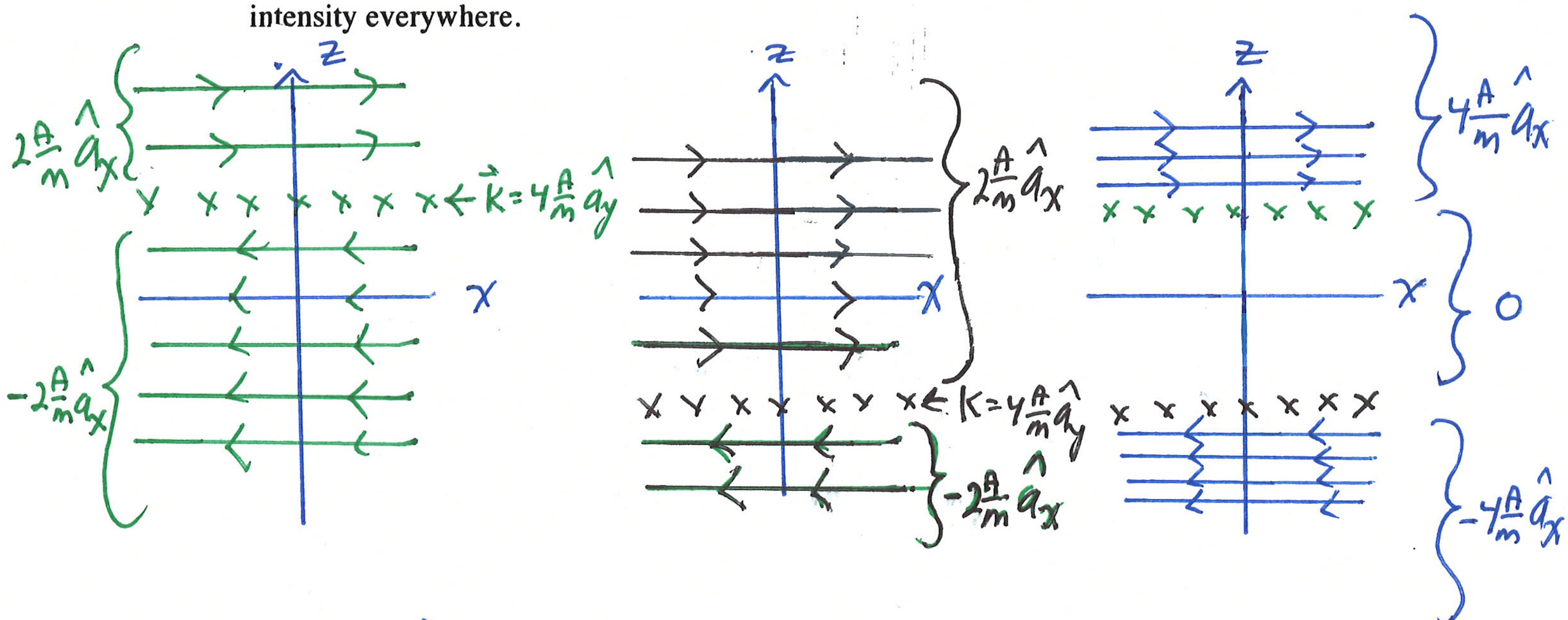








(9 pts) 11. On the z = 1 m plane there is an infinite sheet current density of $\mathbf{K}=4\,\frac{\text{A}}{\text{m}}\,\hat{\mathbf{a}}_\mathbf{y}$ and on the z = -1 m plane there is an infinite sheet current density of $\mathbf{K}=4\,\frac{\text{A}}{\text{m}}\,\hat{\mathbf{a}}_\mathbf{y}$. Find the magnetic field intensity everywhere.



$$\vec{H} = \begin{cases} 4\frac{A}{m}\hat{a}_x & z > 1\,m \\ 0 & -1\,m < z < 1\,m \\ -4\frac{A}{m}\hat{a}_x & z < -1\,m \end{cases}$$